

Inverse engineering and composite pulses for magnetization reversal

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We put forward a method for achieving fast and robust for magnetization reversal in a nanomagnet, by combining the inverse engineering and composite pulses. The magnetic fields, generated by microwave with time-dependent frequency, are first designed inversely within short operation time, and composite pulses are further incorporated to improve the fidelity through reducing the effect of magnetic anisotropy. The high-fidelity magnetization reversals are illustrated with numerical examples, and visualized on Bloch sphere. The influence of damping parameters, relevant to the pulse sequence, is finally discussed based on Landau-Lifshitz-Gilbert equation. These results pave the way for precise but fast magnetization reversal or switching, with the applications in high density information storage and processing.

I. INTRODUCTION

The efficient initialization, manipulation, and readout of electron spins are requisite in the field of spintronics and quantum information for the implementation of high-density information storage and information processing with a single electron spin qubits [1, 2]. Normally, in electric dipole spin resonance (EDSR) [3], microwaves drive an electron to oscillate in the magnetic field or electric fields through spin-orbit coupling, producing a coherent spin manipulation [4–6]. Besides, the Landau-Zener adiabatic schemes [7] provides an analytical model for (effective) two-level quantum systems, achieving the coherent manipulation of spin state [8–10]. However, such adiabatic processes and their variants require long operation time, which becomes inefficient when the damping is considered under decoherent environment. To remedy it, the inverse engineering [11–14] and quantum transitionless algorithm [15, 16], sharing the concept of shortcuts to adiabaticity (STA) [17], and other relevant methods including optimal control [18–20] and composite pulses [21, 22], have been proposed, which mimic the adiabatic control but in accelerated and robust ways.

In ferromagnetic nanostructures, rapid and robust magnetization reversal is of interest for both fundamental physics and applications [23–30]. A large number of experiments have illustrated that the microwave field with appropriate amplitude and frequency in the radio frequency range provides an efficient solution to assist the magnetization dynamics. Mathematically, the problem is somewhat similar to but different from the adiabatic population transfer in atomic two-level systems [27, 29], since the magnetic anisotropy is present. Taking into account the thermal fluctuation or damping parameters, several works pursue the optimal microwave fields for achieve magnetization dynamics, particularly, the magnetization

reversal [25] and switching [26]. But the numerical calculations are sometimes costly, and the optimally chirped microwave fields are cumbersome for practical implementation. In addition, the nonlinear spin dynamics and chaotic behavior induced by magnetic anisotropy or competition between damping and pumping make the magnetization reversal complicated and even disaster [23, 31].

The main purpose of this paper is to combine the inverse engineering and composite pulses for achieving fast and robust magnetization reversal in ferromagnetic nanostructures. The magnetization dynamics of a single-domain uniaxial magnetic particle is described by the Landau-Lifshitz-Gilbert (LLG) equation, in a circularly polarized ac field of constant amplitude but chirped frequency. We first apply the inverse engineering method to design the variable frequency for a given short time and amplitude of the ac field, by assuming the perfect quantum two-level system, and construct the composite pulse to suppress the nonlinear effect resulting from the magnetic anisotropy. But the robustness will be affected by damping parameters, especially when increasing the composite sequence. Finally, the rapid magnetization reversal with high fidelity has been demonstrated with numerical examples and visualized on the Bloch sphere.

II. MODEL AND HAMILTON

We begin with following Hamiltonian, describing the dynamics of a single-domain magnetic particle with uniaxial anisotropy in a circularly polarized ac field,

$$\mathcal{H} = -KVM_z^2 - VM_x h \cos \Phi(t) - VM_y h \sin \Phi(t), \quad (1)$$

where K is the magnetic anisotropy constant, V is the volume of particles, \mathbf{M} is the magnetization, h is the amplitude of the ac field, and $\Phi(t)$ is the phase, producing the time-dependent instantaneous frequency $\omega(t) \equiv \dot{\Phi}(t)$. When the frequency is linearly changing with time, the model resembles the Landau-Zener scheme in conventional quantum (nonlinear) two-level system [27]. Here

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we shall design inversely the nonlinear time-dependent frequency produced by chirped microwave fields for fast magnetization reversal.

In macrospin approximation, by magnetic moment \mathbf{M} , with $|\mathbf{M}| = \mu_s$, the magnetization dynamics is equivalently described by LLG equation,

$$\dot{\mathbf{s}} = \gamma \mathbf{s} \times \mathbf{H} - \alpha \gamma \mathbf{s} \times (\mathbf{s} \times \mathbf{H}), \quad (2)$$

where α is the dimensionless damping coefficient, γ is the gyromagnetic ratio, and the effective Hamiltonian, $\mathbf{H} = -(1/V)\partial\mathcal{H}/\partial\mathbf{M}$, in the rotating frame

$$\mathbf{H} = 2ds_z\mathbf{e}_z + h\mathbf{e}_x + \omega(t)\mathbf{e}_z, \quad (3)$$

with the anisotropy field $d = KM_s$. The initial spin state is antiparticle to \mathbf{e}_z axis which can be prepared by static magnetic field, B_0 , which can be switched off after the initialization. The typical experimental parameters for 3-nm-diameter cobalt nanoparticles are chosen as: gyromagnetic ratio $\gamma = 1.76 \times 10^{11} \text{ T S}^{-1}$, anisotropy constant $K = 2.2 \times 10^5 \text{ J/m}^3$, a volume $V = 14.1 \times 10^{-27} \text{ m}^3$, $M_s = 1.44 \times 10^6 \text{ Am}^{-1}$, and magnetization at saturation $\mu_s = 2.36 \times 10^{-20} \text{ J/T}$ [26, 29]. For the sake of simplicity, we introduce the normalized field $h \equiv h/h_0$ with $h_0 = 2K/\mu_0 M_s \simeq 305 \text{ mT}$, and the dimensionless time corresponds to t/t_0 with $t_0 = 1/(\gamma h_0) \simeq 1.86 \times 10^{-11} \text{ s}$.

To adopt the inverse engineering, proposed in Ref. [11, 12], we first consider the dissipationless problem, when $\alpha = 0$. The LLG equation in general can be parameterized by the convenient spherical coordinates,

$$s_z = \cos \theta, \quad s_x = \sin \theta \cos \varphi, \quad s_y = \sin \theta \sin \varphi, \quad (4)$$

and we obtain, by neglecting the ac field in the dissipation terms, as

$$\dot{\theta} = \gamma h \sin \varphi - \alpha \gamma d \sin 2\theta, \quad (5)$$

$$\dot{\varphi} = -2\gamma d \cos \theta - \omega(t) + \gamma h \cos \varphi \cot \theta. \quad (6)$$

Actually, in presence of magnetic anisotropy, the spin system resembles the nonlinear two-level systems in BEC in double-well potential [32] and accelerated optical lattices [33, 34] and coupled waveguides [35]. One can apply the Eqs. (5) and (6) to engineer inversely the amplitude and frequency of microwave fields, and the time-optimal solution has been obtained accordingly [36]. Nevertheless, stability might be spoiled due to nonlinearity resulting from magnetic anisotropy. Moreover, by assuming that magnetic anisotropic term d is negligible, $d = 0$, the above equations are further simplified as

$$\dot{\theta} = \gamma h \sin \varphi, \quad (7)$$

$$\dot{\varphi} = -\omega(t) + \gamma h \cos \varphi \cot \theta. \quad (8)$$

These are nothing but the auxiliary differential equations, describing the dynamics of population transfer in atomic two-level systems interacting with laser, based on Lewis-Riesenfeld invariant [11] or inverse engineering [13, 14]. Here we shall combine the inverse engineering

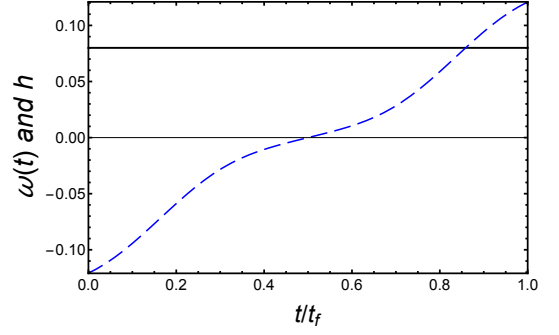


FIG. 1. (Color online) Time-dependent frequency of designed microwave fields versus t/t_0 , where $h = 0.08$ is constant and $t_f = 100t_0$.

and composite pulses, similar to the hybrid method used in Ref. [34]. Our strategy is to apply inverse engineering for linear two-level system to design the frequency of microwave fields for fast magnetization reversal, and further construct the composite pulses by choosing appropriate sequence and phase. This makes the protocol, not only fast but also stable with respect to the variations of experimental parameters, and magnetic anisotropy.

III. INVERSE ENGINEERING AND COMPOSITE PULSES

First of all, we shall design the single pulse of microwave field for speeding up the conventional slowly adiabatic the magnetization reversal by using the inverse engineering method. In principle, the amplitude and frequency of the ac field are both time-dependent and tunable. But the variation of amplitude could be complicated for physical implementation, and could induce the amplitude noise. For simplicity, we consider the Landau-Zener type scheme, that is, the constant amplitude but variant (chirped) frequency. To this end, we rewrite the Eq. (7) by taking the second derivative with respect to time, and obtain

$$\ddot{\theta} = \dot{\varphi} \gamma h \cos \varphi, \quad (9)$$

from which, by combining Eq. (7) and substituting into Eq. (8), we have

$$\omega(t) = -\frac{\ddot{\theta}}{\gamma h \sqrt{1 - \left(\frac{\dot{\theta}}{\gamma h}\right)^2}} + \gamma h \cot \theta \sqrt{1 - \left(\frac{\dot{\theta}}{\gamma h}\right)^2}. \quad (10)$$

This gives the chance to engineer the chirped frequency when the spin trajectory is designed first. But from Eq. (10) the condition, $\dot{\theta} \leq \gamma h$, which implies the operation time t_f should satisfy, $t_f \geq \pi/\gamma h$ ($\approx 40t_0$), for magnetization reversal. Noting that the minimum time for Landau-Zener type scheme is $\pi/\gamma h$, corresponding to

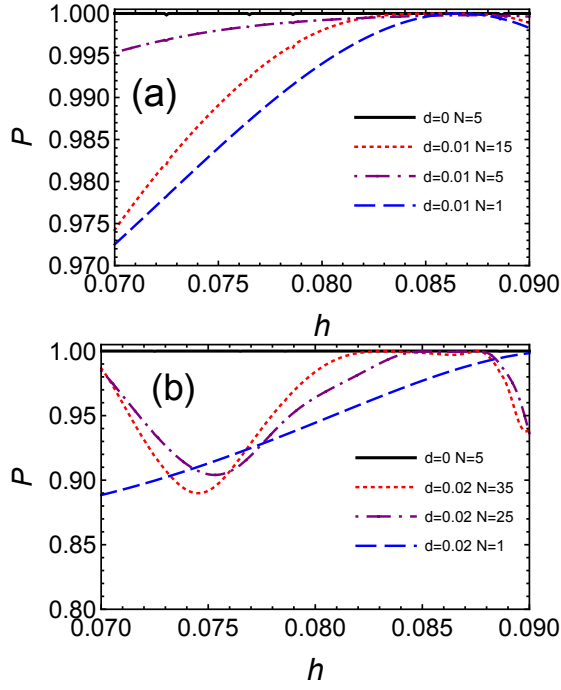


FIG. 2. (Color online) Probability of spin-up state at the final time $t = t_f$ versus the amplitude, h , of microwave fields, with different magnetic anisotropy d and composite sequence N . The parameters are the same as those in Fig. 1.

constant π pulse, and it can be also achieved for unconstrained driving in our system, when $\omega(t) = d \cos \theta$ for cancelling the nonlinearity [36].

Now, we use the inverse engineering method by choosing the following boundary conditions,

$$\theta(0) = \pi, \quad \theta(t_f) = 0, \quad (11)$$

$$\dot{\theta}(0) = -\gamma h, \quad \dot{\theta}(t_f) = -\gamma h, \quad (12)$$

$$\ddot{\theta}(0) = 0, \quad \ddot{\theta}(t_f) = 0. \quad (13)$$

The first two conditions guarantee the magnetization reversal, and others make the frequency smooth at the edges and without singularity. The simple polynomial ansatz $\theta = \sum_0^j a_j t^j$, where the coefficients a_j are analytically solved from boundary conditions. Once θ is interpolated, φ is determined by $\varphi = \sin^{-1}(\theta/\gamma h)$. The chirped frequency is finally designed, from Eq. (10), see Fig. 1, where $t_f = 100t_0$. The advantage of inverse engineering is that the chirped pulse can be designed for a given constant amplitude within short time, as compared to adiabatic control, with Landau-Zener scheme. The shortcut design with only σ_z control is also different from the previous ones presented in Ref. [11] where both Rabi frequency and detuning, corresponding to the amplitude and frequency of field, can be modulated simultaneously.

Next, the composite pulse can reduce significantly the error and suppress the nonlinear effect, even with simple three- and five-pulse composite sequences. Keeping

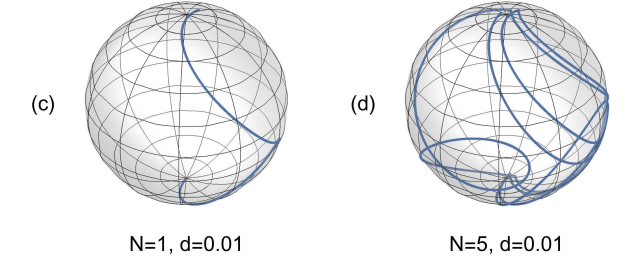
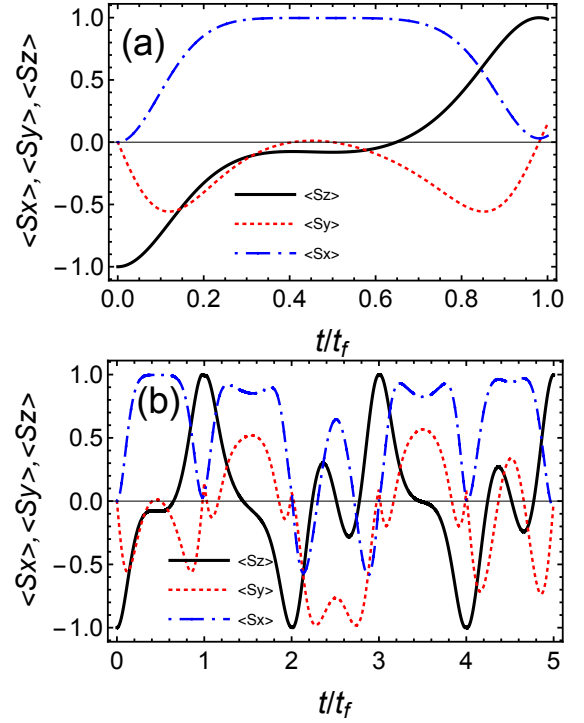


FIG. 3. (Color online) Magnetization dynamics $\langle s_j \rangle$ ($j = x, y, z$) and trajectory on Bloch sphere, where (a,c) $N = 1$ and (b,d) $N = 5$, other parameters are $d = 0.01$, $h = 0.08$, and $t_f = 100t_0$ for each pulse.

this in mind, we construct the composite pulses with a sequence of N ($N = 2n + 1$, n is an integer) pulses, each with a phase ϕ_k ($k = 1, 2, \dots, N$), to achieve high-fidelity quantum control. The phase ϕ_k is imposed on the amplitude of ac field, $h \rightarrow h e^{i\phi_k}$. To shorten the total operation time, we first try shortcut to adiabatic protocol presented above for nonlinear system, the composite control phase in the linear systems is exploited here. In detail, the composite phase is given by [22, 34]

$$\phi_k = \left(N + 1 - 2 \left\lfloor \frac{k+1}{2} \right\rfloor \right) \left\lfloor \frac{k}{2} \right\rfloor \frac{\pi}{N}, \quad (14)$$

where the symbol $\lfloor x \rfloor$ denotes the floor function. The phase sequence is symmetric, i.e., $\phi_k = \phi_{N+1-k}$ and $\phi_1 = \phi_N = 0$.

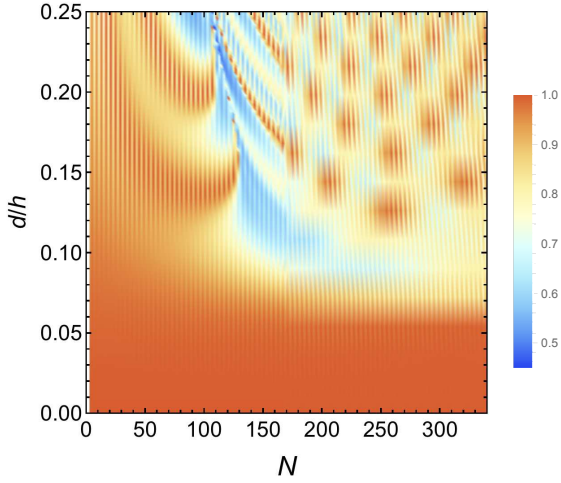


FIG. 4. (Color online) Counterplot of probability of spin-up state at the final time $t = t_f$ versus composite sequence N and magnetic anisotropy d , where $h = 0.05$ and $t_f = 100t_0$.

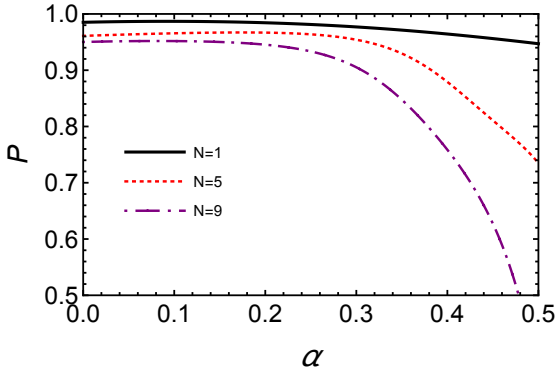


FIG. 5. (Color online) Probability at the final time $t = t_f$ versus the damping parameter α for different composite sequence N , where $t_f = 100t_0$, $h = 0.05$ and $d = 0.005$.

IV. EXAMPLES AND DISCUSSIONS

We are concerned about the magnetization reversal, and thus the probability P of spin-up state at the final time t_f , parallel to \mathbf{e}_z direction. Fig. 2 demonstrates that the perfect magnetization reversal can be achieved by single or composite pulses, when the magnetic anisotropy d is not involved. However, when magnetic anisotropy, d , increasing, the probability is reduced dramatically, and the spin cannot be completely flipped even for large d . Furthermore, the composite pulses even with five composite sequences can improve the probability, taking into account the magnetic anisotropy. For a certain amplitude, h , of the microwave field, the magnetization reversal can be perfect, that is, the probability of spin-up state is almost 1. This result coincides with the nonlinear two-level system [34], implemented in accelerated optical lattice, in which the composite pulse can suppress the nonlinear effect.

In presence of magnetic anisotropy, $d \neq 0$, the magnetization dynamics cannot be described by the parameters θ and φ , since the amplitude and frequency are inversely engineered from Eqs. (7) and (8), in the context of linear two-level systems. Fig. 3 shows the comparison between the cases of a single pulse and five composite sequences. When $d = 0.01$, the magnetization reversal is not perfect by using single pulse, $N = 1$, see Fig. 3 (a). Remarkably, the composite pulse even with five sequences works well, see Fig. 3 (b). The corresponding trajectory of magnetization dynamics are both shown in Fig. 3 (c) and (d). Anyway, by increasing $N = 1$ to $N = 5$, the probability is improved from 0.994 to 0.999. One can also choose large composite sequences, but the probability is not always increased, see the discussion below.

Figure 4 displays the final probability as a function of composite sequence N and magnetic anisotropy d . When the influence of magnetic anisotropy is negligible, the magnetization reversal can be achieved for single and composite pulses. However, the composite pulses with more sequences are required to improve the stability and suppress the nonlinear effect, when magnetic anisotropy increasing. More interestingly, we see from Fig. 4 that the probability at the final time $t = t_f$ oscillates with composite sequence N and magnetic anisotropy d . We identify that the hybrid method combining the inverse engineering and composite pulses has some advantage over the acceleration and stability.

Finally, we turn to discuss the influence of damping parameter α , described in LLG equation (2). The probability at the final time $t = t_f$ for each composite pulses is reduced when increasing the damping parameters. In addition, for a larger composite sequence N , the longer operation time makes the final probability less. In this sense, the damping parameter works as the dephasing noise. One can use the technique of shortcuts to adiabaticity to decrease the time for each pulse, thus avoiding the influence of damping. But the ability of shortening time is in principle limited by the amplitude of microwave fields. So one has to keep the balance by choosing appropriate pulse shape, duration and composite sequence for a given h and d .

V. CONCLUSION

In summary, the method for achieving fast and robust magnetization reversal is proposed in a nanomagnet by combing the inverse engineering composite pulses. The inverse engineering is first applied to design a fast magnetization reversal for each pulse with time-dependent chirped frequency. To suppress the effect of magnetic anisotropy and improve the stability, the composite pulses are further incorporated later. The magnetization reversal with high fidelity have been demonstrated with numerical examples. The influence of field amplitude, magnetic anisotropy and damping parameters are also discussed showing the advantage of hybrid methods.

There are several works for further exploration, for example, the optimization of pulses with respect to different errors and noise [11] and the effect of thermal fluctuations [29]. Besides, alternative approaches, based on Lewis-Riesenfeld invariant, can be tried for the magnetization reversal in the non-equilibrium domain [37, 38]. Last but not least, we hope the fast and robust magnetization reversal or switching can be applicable in high-density information storage and processing in ferromagnetic nanostructures.

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